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DOES LIQUIDITY IMPROVE THE CARHART FOUR-FACTOR MODEL? EVIDENCE FROM THE PAKISTAN STOCK EXCHANGE

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Abstract

This study investigates the explanatory power of the Carhart four-factor model and its liquidity augmented version to explain cross sectional stock returns on the Pakistan Stock Exchange (PSX). The analysis is carried out using a sample of 266 non-financial companies for the period July 2012 to June 2022. Sixteen portfolios based on the firm characteristics were built and the liquidity factor based on the turnover based factor proposed by Datar, Naik and Radcliffe (1998). Time-series regressions, Fama–MacBeth two-pass procedure with Newey–West adjusted standard errors, Gibbons–Ross–Shanken (GRS) and Wald tests were used to evaluate model performance. The empirical results reveal that the traditional Carhart model has little explanatory power, since neither the market factor nor the size, value and momentum factors result in risk premiums that are consistent or statistically significant. The addition of the liquidity factor does not significantly influence the model performance, and the liquidity factor is not significant in both time-series and cross sectional analysis. The results indicate that the Carhart framework does not fully explain the stock return behaviour in the Pakistan Stock Exchange and liquidity is not an additional priced risk factor over the sample period. The study adds to the literature on multifactor asset pricing in emerging markets, and emphasizes the importance of considering alternative sources of systematic risk in future studies.

INTRODUCTION BACKGROUND AND CONTEXT

The goal of investors is to generate higher returns on their investments, and to achieve this objective, they must analyze information pertaining to stocks. However, selecting stocks with the potential for higher returns necessitates the processing of significant amounts of information, which can be a resource-intensive endeavor. Investment decisions, prior to the 1950s, primarily relied on assessing the level of returns without much consideration for risk measurement. However, as the field of finance developed, there emerged a need to quantify risk, leading to the proposal of various models. The Capital Asset Pricing Model (CAPM) has garnered substantial attention and recognition owing to its pragmatic implications. This tool helps investors in constructing portfolios and properly managing their performance.

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Furthermore, the Capital Asset Pricing Model (CAPM) may be used to ascertain the requisite rate of return for investors and the cost of capital for enterprises. The Capital Asset Pricing Model (CAPM) is predicated upon a collection of assumptions, like to other financial theories that provide the foundation for its conceptualization. Although CAPM model did the foundation work for modern asset pricing theory, its implementation in financial market was difficult due to its impractical underlying assumptions especially in emerging markets (Fama & French, 2004; Jagannathan & Wang, 1996). The market imperfections, information asymmetry, limited market depth, transaction costs, and liquidity constraints may impact investors' behavior and asset prices but these effects cannot be captured by a single systematic risk factor (Amihud, 2002; Pastor & Stambaugh, 2003). Hence, the usefulness of CAPM model in markets with structural inefficiencies might be limited which necessitates the development of more comprehensive asset pricing framework (Fama & French, 2015; Thalassinos et al., 2023). Therefore, it is imperative to develop an alternative asset pricing model that integrates more plausible assumptions and combines various elements as explanatory variables. The development of multifactor asset pricing models has been motivated by the acknowledgement that relying on a single risk factor is inadequate in comprehensively elucidating the anticipated returns of an asset.

Since the late 1970s, numerous research studies have emerged, highlighting what are commonly referred to as "anomalies" in relation to the Capital Asset Pricing Model (CAPM). These anomalies challenge the conventional assumptions of the CAPM and shed light on additional factors that influence average stock returns. The development of Single Factor to Multi Factor Asset Pricing Models is a continuous effort to overcome the limitations of the previous models, rather than a chronological development of theories (Carhart, 1997). Each subsequent model was introduced to account for aspects of variation in return that were not previously modeled. However, there is mixed evidence, that the introduction of extra risk factors consistently enhances the pricing accuracy in various institutional and market settings (Barillas et al., 2020; Kostin et al., 2022). This implies, that the applicability of asset pricing models is of a context-specific nature, and needs to be further tested empirically, especially in emerging capital markets.

The research on asset pricing has gradually added more sources of systematic risk to explain the remaining stock return variation that is not captured by previous theories. Of these, the four-factor specification added momentum as a new risk factor for returns, and later research focused on adding liquidity and subsequently profitability, investment, and human capital to the dimensions of systematic risk. These developments are indicative of the ongoing quest for models that can explain the behavior of returns in various market settings. However, it remains an empirical question whether momentum-based and liquidity-based extensions offer additional pricing power, especially in less developed equity markets where institutional conditions vary significantly from those found in developed markets (Carhart, 1997; Pastor & Stambaugh, 2003; Fama & French, 2015; Roy & Shijin, 2018).

The Pakistan Stock Exchange (PSX) is a relevant environment to revisit the asset pricing models as its institutional features are significantly different from the developed financial markets where most of the asset pricing models were first developed and tested (Khan et al., 2023; Thalassinos et al., 2023). Regulatory changes, macroeconomic uncertainty, and other market characteristics, including the relatively illiquid nature of the markets, concentrated ownership structures, information asymmetry, and variable investor participation, can affect the behaviour of the systematic risk factors. All these unique features necessitate the examination of the

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generalizability of conclusions made from developed markets to the Pakistani context pertinent (Rashid & Aib, 2021; Siddiqui et al., 2025).

The momentum and liquidity effects are analyzed in the context of the institutional features of the Pakistan Stock Exchange. In comparison to developed markets, less trading activity, uneven information flow and shallow market depth potentially reduce the persistence of momentum profits and the pricing of liquidity risk. Therefore, the findings of the developed countries should not be extrapolated to the Pakistani market, as it requires an empirical research that is market specific (Pastor and Stambaugh, 2003; Fama and French, 2022; Khan et al., 2023). Although considerable effort has been invested in the development of multi-factor asset pricing models, there is not much empirical evidence on the joint performance of Carhart four-factor model and its liquidity-augmented version in the Pakistan Stock Exchange. The existing domestic research has mainly investigated these factors individually, or analyzed the alternative multi-factors models with a view to excluding liquidity from the analysis, and thus left wide open the question about the incremental impact of liquidity when momentum is added. To address this gap, this study compares the explanatory capability and pricing performance of conventional Carhart model with its liquidity augmented model using the evidence of the Pakistani equity market.

LITERATURE REVIEW

The ability of multifactor traditional asset pricing was challenged by stock with good past performance will continue to perform well and those with poor past performance will continue to perform poorly was a recurring pattern and posed difficulty in capturing the return behaviour fully. The significance of momentum anomaly of Jagadeesh and Titman (1993) motivated Carhart (1997) to add momentum as an additional factor to Fama and French three factor model. This additional momentum factor, represented by winner minus loser portfolio (WML) showing the difference between the return of past good performers (winners) and bad performers (losers). This WML will capture the momentum effect. The inclusion of momentum factor in FF-3FM substantively reduces the pricing error and abnormal returns that remained unexplained under previous asset pricing framework (Jagadeesh & Titman, 1993; Carhart, 1997).

Wermers (1997) and Daniel et al. (2001) has also discovered corroborative evidence pertaining to Carhart's fourth factor. The factor focuses on the various techniques that facilitate persistence in mutual fund. Tai (2003) explores the impact of FF three common risk factors, in addition to the momentum factor introduced by Carhat (1997), on the returns of securities. The researcher employed monthly data from NYSE, AMEX and NASDAQ, spanning from June 1953 to May 2000. The findings of Tai's revealed that all risk factors, namely market return (RM-RF), size (SMB), book-to-market (HML), and momentum (MOM), are significantly priced and reflected in the rate of return of stocks. The outcomes of the study demonstrated that these risk factors offer a superior explanation for the time-series variation in stocks return, relative to the CAPM. L'Her et al. (2004) undertook an investigation of the four factor model proposed by Carhart (1997) to provide enhanced explanatory power for the time-series fluctuations in stock returns over Fama and French's three factor model. This investigation utilized monthly data obtained from the Canadian stock market spanning the years 1960 to 2001. The authors report the presence of robust evidence in favor of the four factor model. The findings of the Bello (2008) also proved that the four factor model outperformed the FF-3 and CAPM. The study utilized the equity mutual funds situated in the United States of America during the period ranging from April 1986 to March 2006.

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Awwaliyah and Husodo (2011) conducted a test using US data from January 1963 to December 2010 to assess the validity of the three-factor Fama-French and four-factor Carhart models. Their findings indicated that the four-factor Carhart model outperformed the Fama-French three-factor model in explaining the variation in excess returns of stock portfolios in the US. Fama and French (2012) examined the impact of size, value, and momentum in 23 developed countries across four regions: North America, Japan, Asia Pacific, and Europe. Their study covered data from November 1989 to March 2011. The results revealed the existence of a value premium and return momentum in all regions, except for Japan. Empirical studies suggested that by extending the traditional asset pricing models with the momentum factor, the ability of the model to explain cross-section of asset returns is further improved and momentum is shown to predict return persistence beyond that measured by the market, size and value factors. However, the strength and consistency of the momentum premium vary between countries and market environments, indicating that the momentum premium may be context dependent in pricing ability.

The four factor model helps explain the variation in returns but there is mixed evidence on whether it explains all pricing anomalies in emerging markets. Tai (2003) reported large momentum effects in the U.S. market, while Fama and French (2012) found that momentum effects appear to be weaker in certain markets than others, including Japan. Fama & French (2022) indicated that momentum premiums are not always present in the same market conditions, especially in less-developed and emerging markets where market frictions and institutional factors can affect the return dynamics. The Carhart four-factor model has been supported by empirical studies, but the results are inconsistent in emerging markets where market structure and trading conditions vary from those in developed markets.

Liu (2006) defined liquidity as "liquidity is how easily a stock can be traded without compromising on its price and volume with no trading cost." The definition guides investors to do their analysis thoroughly before investment in an asset that if they want to sell it in future what cost and price they have bear. All these liquidity related issue affects its price, thus making it an important factor in pricing decision. The empirical evidence on the role of liquidity in pricing is inconclusive. Datar, Naik and Radcliffe (1998) reported that less liquid stocks have higher average returns, which implies investors should be compensated for taking liquidity risk. Likewise, research by Pastor and Stambaugh (2003) found that liquidity was a systematic risk factor in the expected return. Some recent evidence still indicates mixed results on the pricing of liquidity risk. Some studies indicate that liquidity factors enhance the explanation of expected returns (Iqbal & Shahzad,2024; Jain & Singla,2022; Carvalho et al., 2021) especially in times of market stress, while others indicate that once conventional risk factors are controlled for, liquidity factors have little impact (Fama & French, 2016; Saeed et al.,2020). From empirical evidence, the importance of liquidity is seen to differ between developed and emerging markets based on market structure, trading activity, and the type of liquidity used. However, inconsistent results suggest that the contribution of liquidity is still an empirical matter, especially for emerging equity markets. Furthermore, the evidence of the incremental explanatory power of liquidity over the Carhart framework is still limited, especially in the context of Pakistan Stock Exchange. This unresolved issue motivated the present study which examines the pricing performance of the conventional Carhart model and its liquidity-augmented specification in the Pakistani equity market.

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HYPOTHESIS

H1: there is systematic pricing of market, size, value and momentum factors and together they increase the explanatory power of asset pricing in framework of Carhart-4FM.

This is a hypothesis used to assess the existence of momentum as a factor that helps to explain excess stock returns in addition to market, size and value factor. The Carhart four-factor model uses the momentum to document persistence in the performances of stocks which are not accounted by the traditional risk factors (Carhart, 1997). Empirical research on both developed and emerging markets report existence of momentum effects. Momentum has the potential to increase predictability of returns considering behavioral trends in trading and information inefficiency in PSX. Thus the ability to explain the asset pricing model using the momentum consideration will likely to enhance its explanatory strength.

The econometric model that will test the hypothesis is as follows.

$$-= +{}_1(MarErt) + {}_2 (+{}_3 +{}_4) + \quad (\quad) \quad (\quad)$$

H2: The liquidity risk factor is statistically significant and it improves the explanatory power of Carhart four factor model in explaining excess returns.

The hypothesis will test the existence of liquidity risk being a systematic determinant of excess stock returns. A great deal of empirical research showed that the price of liquidity has to be compensated due to the necessary consideration of the trading frictions and liquidity risk among investors (Amihud, 2002; Acharya and Pedersen, 2005). The behavior of the stock returns and the improvement of the model can also be performed by evidence of the Pakistan Stock Exchange which indicated that liquidity factors have effects on stock returns (Haqqani and Aleem, 2020; Azam, 2023). Since the market depth in PSX is comparatively low, the transaction costs and trading frictions are high, the liquidity risk is expected to be more intense.

The hypothesis will be tested under the estimation model of Carhar-4FM. The econometric models that will test the hypothesis is

$$-= +{}_1(MarErt) + {}_2 (+{}_3 +{}_4) +{}_5L +) \quad (\quad) \quad (\quad)$$

METHODOLOGY

The study sample frame was comprised of all non-financial company listed on the Pakistan Stock Exchange (PSX) for the period July 2012 to June 2022. Financial firms were removed off the sampling frame as they have a different regulation framework and financial reporting requirements that are very different compared to non-financial firms and a different regime would introduce comparability problems in the analysis of asset prices.

In order to assure the reliability of findings and consistency in developing factor, it was restricted to firms who had monthly return information and annual accounting information for the whole sample period i.e. from 1st July 2012 to 30th June 2022. Companies that has any missing values or incomplete financial statements were not part of the research. Total no of non-financial companies in sample after screening is 266.

This study utilized secondary data collected from various credible sources. To calculate the stock returns during the sample period between July 2012 and June 2022, monthly data of stock price and market capitalization of non-financial companies listed in Pakistan Stock Exchange (PSX) were obtained from the website www.opendoors.pk. The three months yield of the Treasury bills were obtained from State Bank of Pakistan (SBP) report that was used as the risk-free rate. Whereas the market return data were obtained from the website www.investing.com, using the benchmark PSX 100 index data. Accounting variables annual data at the firm level such as the book value of equity and total assets were obtained from the annual reports of the companies.

Size, value, profitability and investment were used to sort portfolios (2×2×2×2). The process led to the emergence of sixteen portfolios.

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Portfolios were rebalanced at every financial year closing in order to avoid look ahead bias.

The SMB and HML factors were constructed by subtracting the equal weighted average returns of big and low stock from equal weighted average returns of small and high stocks respectively.

Construction of momentum factor by following the Carhat (1997) which is "Past Winner-Past loser". For each stock calculated "12-1 (t-1)" returns every year to avoid the reversal bias. Then rank the stocks on basis of their returns from lowest to highest. By using the 30-70 percentile sorting, that is, the stocks which lied in the top 30% of ranking was tagged as "loser" while the bottom 30% as "winner". The winner portfolio was constructed by taking the equal weighted average of return of winner stock while loser portfolio represents the equal weighted average of returns of loser stocks. Now at time "t" the momentum factor is

$$M = tr - r$$

The same process was repeated every year from July 2012 to June 2022.

Liquidity factor was constructed by following methodology of Datar, Naik and Radcliffe (1998). Turnover is employed as a proxy to liquidity as it represents the trading activity of a particular stock and is extensively used in empirical aspects of assets pricing mainly in emerging markets. Share Turnover that was calculated for each stock were then sorted from lowest to highest. The median was taken to divide the stocks into two groups which are low and high turnover. Stocks that has low turnover ratios represents Illiquid while the high turnovers shows liquid stocks. Now the liquidity factor was estimated by subtracting the equal weighted average of returns of illiquid stocks from equal weighted average returns of liquid stocks.

$$L = ei - ei$$

The explanatory power of competing asset pricing models was investigated using the time-series regression analysis method. Factor loading and intercept estimations were made as part of Ordinary Least Squares (OLS) estimation. The coefficients and adjusted R^2 values were tested to compare model performance. Intercept term (alpha) was tested to determine whether the portfolio returns were completely explained by the risk factors identified, or not. The two-pass regression methodology introduced by Fama and MacBeth (1973) was used to test the extent to which the risk factors identified were priced in the cross-section of returns. Newey-West adjusted standard errors were used to assess statistical significance of the estimated Fama-MacBeth risk premia. Gibbons, Ross and Shanken (1989) test was used for joint testing of whether the intercepts are equal to zero. The Wald test was employed to test the joint significance of the co-efficient in the augmented models. This test enabled a comparison across the specifications of the models and also helped to find out whether inclusion of liquidity factors led to improvement of model performance.

RESULTS

TABLE 1(a)Time-Series CARHART 4FM

Portfolio	Intercept	MKT	SMB	HML	WML	R2	Adj_R2	F_stat	Prob(F)
BHRA	0.06	0.553	-1.283	-0.369	1.33	0.302	-0.257	0.54	
	0.2053	0.6973	0.4319	0.7722	0.3763				0.7147
BHRC	0.043	0.367	-0.761	0.713	0.49	0.692	0.446	2.813	

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	0.0383**	0.49	0.2259	0.1687	0.3752				0.1433
BHWA	0.103	0.504	-2.206	-0.599	1.567	0.43	-0.026	0.943	
	0.0994*	0.7722	0.2854	0.7028	0.3949				0.5091
BHWC	0.151	1.742	-3.752	-1.945	3.397	0.633	0.34	2.159	
	0.0267**	0.3191	0.0865*	0.228	0.0884*				0.2104
BLRA	0.08	0.574	-1.697	-0.259	1.236	0.585	0.254	1.766	
	0.0300**	0.5344	0.138	0.7509	0.2184				0.2725
BLRC	0.076	1.052	-2.037	-0.753	1.466	0.351	-0.168	0.677	
	0.1083	0.44	0.2065	0.5341	0.3045				0.6365
BLWA	0.089	1.939	-3.274	-1.562	2.411	0.38	-0.116	0.767	
	0.1651	0.322	0.1582	0.3701	0.2386				0.59
BLWC	0.14	0.955	-3.076	-1.173	2.206	0.48	0.065	1.155	
	0.0571*	0.6262	0.1956	0.5102	0.2937				0.4285
SHRA	0.093	1.181	-1.93	-0.622	1.752	0.454	0.017	1.039	
	0.0332**	0.3057	0.1566	0.5338	0.1585				0.4704
SHRC	0.137	0.762	-0.521	3.013	1.864	0.489	0.079	1.194	
	0.1547	0.7851	0.8674	0.2615	0.5216				0.4156
SHWA	0.088	1.398	-1.8	-0.874	2.197	0.388	-0.102	0.792	
	0.0864*	0.3426	0.2817	0.4988	0.1678				0.5775
SHWC	0.066	1.178	-1.834	-1.265	1.507	0.237	-0.373	0.388	
	0.2027	0.4555	0.3117	0.3787	0.3577				0.8098
SLRA	0.07	1.208	-0.698	-1.602	1.42	0.433	-0.021	0.955	

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	0.108	0.348	0.6154	0.187	0.2867				0.5041
SLRC	0.062	0.549	-1.419	-1.443	1.298	0.202	-0.436	0.317	
	0.3276	0.7787	0.5238	0.4257	0.5229				0.8561
SLWA	0.095	0.824	0.046	-2.479	2.086	0.615	0.308	2	
	0.1063	0.6218	0.98	0.1389	0.2494				0.2328
SLWC	0.129	0.586	-1.931	-0.676	1.981	0.425	-0.035	0.924	
	0.0593*	0.747	0.3615	0.6802	0.3105				0.5172

TABLE 1(b)FAMA-MACBETH

*** p < 0.01

Factor	Mean(λ)	NW_t	p_value
Intercept	0.028614**	2.835604	0.02196
MKT_RF	0.0151	0.808773	0.442032
SMB	0.008474	0.647317	0.535556
HML	-0.0067	-0.84017	0.425211
WML	-0.00388	-0.25414	0.805795

TABLE 1(c) GRS AND WALD TEST

Model	Observations	GRS	GRS_p
Carhart_4FM	120	2.121619	0.012709

Table 1(a) shows the systematic pricing of market, size, value and momentum factors and their joint effect on the explanatory power of asset pricing in framework of Carhart-4FM. The intercept of all portfolios are insignificant suggesting the presence of abnormal returns specifically in large

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portfolios and few small aggressive portfolios. When momentum factor is included, the SMB and HML coefficients become statistically insignificant in all portfolios. The momentum factor itself is weakly priced in the context of PSX, implying that the systematic momentum premium is non-existent during the sample period. The results are same as Fama and French (2022), where they revealed the weak performance of momentum factor in emerging markets. The market factor is also weak and exhibits limited significance that supports the dominance of systematic beta risk.

The small values of adjusted R square even negative shows the poor performance of momentum augmented FF-3FM. All portfolios carry F-statistics greater than 0.14, implying the weak fit of model suggesting that the inclusion of momentum factor does not increase the explanatory power of base asset pricing model.

Table 1(b) shows the findings of Fama-MacBeth that all the risk factors, including market, size, value, and momentum, do not have statistically significant cross-sectional risk premium. The intercept is positive and significant, implying that there is systematic pricing errors. GRS shows that the Carhart model does not fully explain the variation in the portfolio returns and complex pricing error are yet to be explained. Hence, the base Carhart model fails to illustrate cross-sectional price power in the sample and model efficiency is rejected in terms of the GRS test.

TABLE 2(a) Time-Series LIQ-Carhart-4FM

Portfolio	Intercept	MKT	SMB	HML	WML	LIQ	R2	Adj_R2	F_stat	Prob(F)
BHRA	0.069	0.073	-1.515	-0.476	1.445	-0.009	0.368	-0.422	0.466	
	0.2094	0.9662	0.4072	0.7319	0.381	0.553				0.7882
BHRC	0.041	0.479	-0.708	0.737	0.463	0.002	0.704	0.334	1.902	
	0.0799*	0.4764	0.3173	0.2068	0.4517	0.7125				0.2765
BHWA	0.115	-0.18	-2.536	-0.751	1.73	-0.013	0.502	-0.12	0.808	
	0.1073	0.9306	0.2685	0.6556	0.3843	0.488				0.5986
BHWC	0.162	1.093	-4.065	-2.089	3.552	-0.013	0.679	0.278	1.694	
	0.0375**	0.5876	0.0977*	0.2343	0.1041	0.4913				0.3149
BLRA	0.081	0.508	-1.729	-0.274	1.252	-0.001	0.587	0.071	1.138	
	0.0587*	0.6631	0.1911	0.7685	0.2735	0.9016				0.463
BLRC	0.085	0.525	-2.291	-0.87	1.592	-0.01	0.436	-0.269	0.618	

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	0.1142	0.7398	0.2014	0.5039	0.3025	0.4822				0.6977
BLWA	0.086	2.092	-3.201	-1.528	2.375	0.003	0.384	-0.387	0.498	
	0.2461	0.3991	0.2291	0.4399	0.3059	0.8885				0.7685
BLWC	0.162	-0.293	-3.678	-1.45	2.503	-0.024	0.656	0.227	1.529	
	0.0399**	0.8842	0.1278	0.3933	0.2197	0.2254				0.351
SHRA	0.101	0.724	-2.15	-0.723	1.861	-0.009	0.533	-0.052	0.911	
	0.0426**	0.5812	0.1549	0.4977	0.1666	0.4583				0.5512
SHRC	0.162	-0.701	-1.226	2.688	2.212	-0.028	0.604	0.108	1.218	
	0.125	0.8233	0.7027	0.3188	0.4559	0.3417				0.4363
SHWA	0.084	1.626	-1.69	-0.823	2.143	0.004	0.401	-0.348	0.536	
	0.1521	0.3837	0.3742	0.5714	0.2309	0.7817				0.7457
SHWC	0.066	1.183	-1.832	-1.263	1.506	0	0.237	-0.717	0.248	
	0.2772	0.5545	0.3823	0.4404	0.4198	0.9955				0.9205
SLRA	0.069	1.315	-0.647	-1.578	1.394	0.002	0.437	-0.268	0.62	
	0.1763	0.4204	0.6867	0.2514	0.356	0.8819				0.6967
SLRC	0.064	0.43	-1.477	-1.469	1.327	-0.002	0.205	-0.79	0.206	
	0.3897	0.8627	0.567	0.4767	0.5665	0.9169				0.9435
SLWA	0.104	0.319	-0.197	-2.591	2.206	-0.01	0.645	0.201	1.453	
	0.1274	0.8748	0.9236	0.1649	0.2719	0.5957				0.3696
SLWC	0.15	-0.599	-2.503	-0.939	2.263	-0.023	0.627	0.16	1.343	

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	0.0398**	0.7489	0.2315	0.5401	0.2285	0.2155				0.399

TABLE 2(b) FAMA-MACBETH

Factors	Mean(λ)	NW_t	p_value
Intercept	0.025595**	2.674857	0.02815
MKT_RF	0.032407	1.771465	0.11443
SMB	0.012533	1.019755	0.33769
HML	-0.00611	-0.76175	0.46807
WML	-0.00921	-0.59126	0.570668
LIQ	-3.13219*	-1.88538	0.096107

*** p < 0.01

TABLE 2(c) GRS AND WALD TEST

Model	Observations	GRS	GRS_p	Wald	Wald_p
Carhart_LIQ_5FM	120	2.06297	0.015926	0.404197	1

Liquidity under Carhart-4fm

Table 2(a) shows the impact of liquidity on Carhar-4FM. After adding the momentum and liquidity effects, all the traditional effects, size (SMB) and value (HML) are relatively insignificant across the portfolios, indicating that their explanatory power does not stand in the Carhart-4FM. Even the momentum factor does not show the presence of a consistent statistical significance, that is lack of the continuation premium of returns throughout the sample. Also, the liquidity factor has largely insignificant coefficients in different portfolios, implying that liquidity risk does not attract systematic premium even when the effects of momentum are considered. The F-statistics, which proves the irrelevance of the joint models and indicates that the overall explanatory power is not influenced by the newly added risk dimensions but represents the overlapping of a factor or the poor ability of a factor. These results are consistent with recent international data that there are numerous instances of momentum and liquidity premiums that are weak, time-varying, or vanish in emerging markets when traditional risk variables are factored out (Fama & French, 2022). Hence, the liquidityaugmented Carhart specification lacks significant incremental

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explanatory power, which means that neither the momentum nor the liquidity risk are price in the PSX in the sample period.it is also evident from values of R-square.

Table 2(b) shows the results of Fama-Macbeth second pass that confirms liquidity factor cannot enhance cross-sectional pricing. All of the systematic risk factors, such as momentum and liquidity, do not receive statistically significant risk premium. The large value of the intercept implies the existence of unexplained average returns which implies that the model is mis-specified. Liquidity does not turn out a priced factor of the crosssection. The Wald test from Table 4.10(c) does not reject joint null of zero alphas, but the GRS rejects the model efficiency at 5% level of significance.

Thus, liquidity-augmented Carhart model fails to provide improvement in pricing performance, and pronounced pricing mistakes remain.

CONCLUSION

This study aimed to investigate the validity and performance of Carhart four factor and its augmented asset pricing model in explaining the cross sectional stock returns in the Pakistan Stock Exchange. This was achieved by formulating sixteen portfolios based upon the firm characteristics such as size, book-to-market ratio, profitability, and investment which capture major aspects of systematic risk advanced in contemporary asset pricing research. In addition, the Carhart framework was further enhanced by incorporating new factor liquidity to investigate whether there is incremental impact of this new risk dimension in an emerging market setting. The empirical study used time-series regression model, the Fama-MacBeth two-pass model and tests of model evaluation: Gibbons-Ross-Shanken (GRS) and Wald in order to determine pricing accuracy, contribution and the relative effectiveness of rival specifications.

The results suggest that Carhart four factor model has limited explanatory power over the portfolio returns for the Pakistan Stock Exchange during the sample period. Overall, the market, size, value and momentum factors provide little consistent and statistically significant risk premiums and pricing errors persist, indicating that there are significant sources of return variation that are not captured. In overall conclusion, the evidence suggests that conventional Carhart model is not enough to explain the cross sectional dynamics of stock returns in the Pakistani equity markets. The empirical evidence gives a weak evidence of liquidity as a systematic determinant of returns in the market analysed. The time-series regressions show that exposure to liquidity risk is low in the constructed portfolios, and the factor loadings are insignificant in most cases. It means that portfolio return behavior does not necessarily depend on variations in trading frictions or market liquidity. This observation is also supported by the crosssectional estimates generated using the FamaMacBeth method, where the liquidity premium estimate does not become statistically significant. This implies that the returns on the portfolio under consideration in the sample period do not consistently reward investors to bear the liquidity risk.

Theoretically, the results confirm the opinion that emerging market asset pricing relation might not be the same as that of the developed markets. Weak and unstable prices of various factors are indicative of the possibility that market imperfections, structural constraints and informational inefficiencies are a greater determinant of returns. Consequently, empirical evidence of this study can be added to the knowledge of current asset pricing models that offer a partial explanation of return behaviour in the Pakistan stock Exchange and that more research is needed to determine other sources of systematic risk. The results indicated that there is a need for further adaptation of traditional multifactor asset pricing models to better explain the return behavior in emerging equity markets like Pakistan.

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REFERENCES:

Amihud, Y. (2002). Illiquidity and Stock Returns: Cross-Section and Time-Series Effects. *Journal of Financial Markets, 5*(1), 31-56.

Amihud, Y. & H. Mendelson (1986), "Asset pricing and the bid-ask spread". *Journal of Financial Economics 17*, 223-249.

Awwaliyah, I. N., & Husodo, Z. A. (2011). On the robustness of the extended Fama-French three factor model. Unpublished Working Paper. Universitas Indonesia.

Bello, Z. Y. (2008). A statistical comparison of the CAPM to the Fama-French Three Factor Model and the Carhart's Model. Global Journal of Finance and Banking Issues, 2(2).

Carhart, M. M. (1997). On Persistence in Mutual Fund Performance. The Journal of Finance, 52(1):57–82.

Datar, V. T., Naik, N. Y., & Radcliffe, R. (1998). Liquidity and stock returns: An alternative test. Journal of Financial Markets, 1(2), 203–219.

Fama, E. F., & French, K. R. (1993). Common risk factors in the returns on stocks and bonds. Journal of Financial Economics, 33, 3–56.

Fama, E. F., & French, K. R. (2012). Size, value, and momentum in international stock returns. Journal of Financial Economics, 105, 457–472.

Fama, E. F., & French, K. R. (2015). A five-factor asset pricing model. Journal of Financial Economics, 116, 1–22.

Fama, E. F. and French, K. R. (2016). Dissecting anomalies with a five-factor model. Review of Financial Studies, 29(1):69–103.

Jegadeesh, N. and Titman, S. (1993). Returns to Buying Winners and Selling Losers: Implications for Stock Market Efficiency. The Journal of Finance, 48(1):65–91.

L'Her, J.-F., Masmoudi, T. & Suret, J.-M. (2004). Evidence to support the four-factor pricing model from the Canadian stock market. International Financial Markets, Institutions and Money, 14, 313-328.

Tai, C. S. (2003). Are Fama–French and momentum factors really priced?. Journal of Multinational Financial Management, 13(4-5), 359-384.

Iqbal, S and Shahzad, N. (2024). A Liquidity-Augmented Capital Asset Pricing Model: Analytical and Empirical Perspectives. Dialogue Social Science Review (DSSR), 2(5), 267–281.